**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

In the problem given μ = 45 , σ = 8

the work begins 10 mins after the car is dropped, the time left to complete work is 50 minutes

Probability that service manager cannot meet his commitment = P(x>50) = 1- P(x<=50)

Here x is the time taken to complete the work

Convert to 50 to z- score

Standard normal variable Z = (X-*μ*)/*σ*

# *=(x-45)/8*

# *= (50-45)8*

# = 0.625

# Z -calculated value = 0.625

# Then in the z -table the value is 0.73232

# *PR=(z<=0.625) = 0.7323 = 73.237%*

# *Probability that service manager will not meet his commitment is 100-73.237 = 0.2676*

# *So the answer is B. 0.2676*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANSWER : *μ* = 38,*σ* =6

# A: probability of employee >44=P(x<44)=1-P(x<=44)

# Z=(X-*μ)/σ*

# *=(X-38)/6*

# =P(44-38)/6

# = P(1)

# =0.84134=84.134%--> z calculated value in the z table .

# Probability that employees will be greater than 44.

# =100-84.134=15.866%

# Probability of employees between 38 and 44 = P(x<=44)-P(x>=38)

# P(x>=38)=P(38-38)/6

# P(x>=38)=P(0)

# =P(0) 🡪 z value is in the table 0.5% =5

# Therefore , the probability of employees between 38 and 44 is

# (x<=44)-P(x>=38)

# =0.84134-0.5= 0.34134=34.134%

Conclusion:

# So, the statement “more employees at the processing center are older than 44 betweem 38 and 44 “ is True.

B)

# Probability of employees less than the 30 = P (x<30)

# Z=( X-*μ)/σ*

# P(x<30)=P(z<(30-38)/6)

# = P(z<-1.3333)

# =.09176=9.17% 🡪 z calculated value is 9.176.

# So, the no.of employees with probability 0.0917 of them being under30=400\*0.0917=36.68=36

Conclusion:

# The statement of “Training program for employees under the age of 30 at the center would be expected to attract about 36 employees” is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

# In the given problem,

# If *X1* ~ *N*(μ, σ2)

# *X*2 ~ *N*(μ, σ2) these are two independent random variables .

# 2x1:-

# 2x1~N(2 μ1,4 σ2)

# X1+X2:-

# X1+X2~N(μ1~ μ2, σ2 +σ2)~N(2 μ,2 σ2)

# 2X1~(X1+2X1) = N(4 μ,6 σ2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

# ANSWER:

# Probability of getting value between a and b is 0.99.

# So , the probability getting outside value a and b is = 1-0.99 = 0.01

# Probability towards left of a = -0.01/2 = -0.05

# Probability towards right of b = 0.01/2 = 0.05

# By finding the Standard Normal Variable (z) , need to calculate X:

# Z = (X-Mue)/Sigma

# For a probability of 0.005 , z value is -2.57

# Z\* σ+ μ=x

# -(-2.57)\*20+100=151.4

# (-2.57)\*20+100=48

# Option D is correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

# Let P be the range such that it continuous 95% probability for the annual profit of the company

# P(P<p) = 95%

# P((P – 12) / Sqrt 74 < (p -12) / Sqrt 74)

# P – 12 / Sqrt 74 ~ N(0.1)

# Pie(x) (p – 12) / Sqrt 74) = 0.95

# Where Pie(x) represented the CDF of a Standard normal random variable

# From the standard normal distribution table it is obtained that

# Pie((p – 12) / Sqrt74) = 0.95

# p-12 / Sqrt 74 = 1.644854

# p = 12 + 1.644854 \* Sqrt 74

# p = 13.415

# p~$13.415 = Rs 45 \* 13.415

# = Rs 603.68

# P(P < q) = 5%

# P((P – 12) / Sqrt 74 < (q – 12) / Sqrt74)

# = 0.05

# P – 12 / Sqrt 74 ~ N(0,1)

# Pie(q – 12 / Sqrt 74)

# = 0.05

# Note: -

# The standard normal distribution table it is obtained that

B . Which of the two divisions has a larger probability of making a loss in a given year?

# Pie(p – 12) / Sqrt 74) = 0.05

# p – 12 / Sqrt 74 =

# -1.644854 \* Sqrt 74

# = 10.585

# P ~ $10.585

# = Rs 45 \* 10.585

# = Rs 476.33

C . Which of the two divisions has a larger probability of making a loss in a given year?

# Probability of first division making a loss = P(Profit 1 < 0)

# Profit ~ N(5, 32) = Profit 1 – 5 / Sqrt 32 ~ N(0,1)

# P(Profit 1 < 0) = P(Profit 1 – 5 / Sqrt 32 < 0 – 5 / Sqrt 32)

# = P(Profit 1 – 5 / Sqrt 32 < -0.88388)

# = 0.18838

# Note: -

# The above probability is obtained from the standard normal distribution table

# Probability of second division making a loss = P(Profit 2 < 0)

# Profit 2 ~ N(7,42) = Profit – 7 / Sqrt 42

# P(Profit 2 < 0) = P(Profit 2 -7 / Sqrt 42 < 0 – 7 / Sqrt 42)

# = P(Profit 2 – 7 / Sqrt 42 < -1.08012345

# = 0.140044

# Note: -

# The above probability is obtained from the standard normal distribution table.

# Thus , the first division has a larger probability of making a loss in a given year.